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LETTER TO THE EDITOR

Thermal conductivity in a partially degenerate electron plasma

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Abstract. The dielectric RPA response function is used to evaluate the thermal conductivity of a partially degenerate electron gas in the presence of classical ions, for any temperature.

With the feasibility of laser driven controlled fusion, one is faced with the new challenging problem of evaluating the thermal conductance of the partially degenerate electron plasma enclosed in the inner compression core (Brueckner and Jorna 1974). This quantity is of a paramount importance in determining the quantity of laser-deposited energy reaching the reacting zone. In this work, we address ourselves to the realistic approximation which consists of considering the compression plasma in thermal equilibrium with partially degenerate electrons deflected by massive classical ions. Thus neglecting any turbulent heating, the thermal conductivity is expected to be controlled by the light electrons at high temperature ($\sim 10^7$ K), in such a high-density regime ($n_e \sim 10^{27}$ cm^{-3}) that the Pauli principle becomes non-negligible ($k_B T_e \leqslant \mathscr{E}_F$). A similar problem has already been considered in an astrophysical context (Lampe 1968a,b), where the departure from the Lorentz model due to the electron–electron interaction has been worked out in the long-wavelength limit of the static RPA dielectric function at $T_e = 0$. In this work, we take advantage of a new dielectric RPA $\epsilon(q, \omega)$ worked out for any degeneracy ratio $k_B T/\mathscr{E}_F$ (Gouedard and Deutsch 1977), and the above defined two-component plasma as

$$\mathscr{E}_{\mathrm{RPA}}(q,\omega) = \frac{1+4\pi e^2}{q^2}\left(\frac{4\alpha r_s^e}{\pi}\frac{k_F^{e2}}{4\pi e^2} g^e(Q_e, \nu_e) + \frac{4\alpha r_s^i}{\nu}\frac{k_F^{i2}}{4\pi e^2} Z^2 g^i(Q_i, \nu_i)\right) \tag{1}$$

where

$$g(q,\omega) = \frac{1}{2Q}\left(f\left(\frac{\nu}{2Q}+\frac{Q}{2}\right) - f\left(\frac{\nu}{2Q}-\frac{Q}{2}\right)\right)$$

$$\operatorname{Re} f(x) = 2x\left\{\int_0^\infty \frac{\mathrm{d}k}{\exp(\beta\hbar^2 k^2/2m)+1} + \pi T\sum_{n=0}^{\infty}\frac{b_n}{r_n^2} - \frac{1}{2x}\left[\tan^{-1}\left(\frac{x+a_n}{b_n}\right) + \tan^{-1}\left(\frac{x-a_n}{b_n}\right)\right]\right\}$$

$$\operatorname{Im} f(x) = \tfrac{1}{2}\pi T \ln[1+\exp(\gamma - x^2/T)]$$

and

$$Q = q/k_F, \qquad \nu = \hbar\omega/\mathscr{E}_F, \qquad \gamma = \mu/k_B T, \qquad T = k_B T/\mathscr{E}_F,$$
$$2a_n b_n = (2n+1)\pi T, \qquad a_n^2 - b_n^2 = \gamma, \qquad r_n^2 = a_n^2 + b_n^2,$$

$\alpha = (9\pi/4)^{-1/3}$ and r_s is the interparticle distance in units of a_0.

Now, let us introduce the time-independent transport quantities through the Onsager relationships

$$\begin{aligned} \boldsymbol{J} &= eG_{11}\left(e\boldsymbol{E} + \frac{\nabla P}{n}\right) + eG_{12}\frac{\nabla T}{T} \\ \boldsymbol{q} &= -G_{21}\left(e\boldsymbol{E} + \frac{\nabla P}{n}\right) - G_{22}\frac{\nabla T}{T} - \frac{5}{3}\frac{\boldsymbol{J}}{e}\mathscr{E} \end{aligned} \tag{2}$$

where the convective heat flux has been singled out and $\mathscr{E}$ is the mean electronic kinetic energy. The thermal conductivity at zero current then reads

$$K = \frac{G_{11}G_{22} - G_{12}^2}{TG_{11}} \tag{3}$$

with (m = electron mass)

$$\begin{aligned} G_{11} &= \frac{2\hbar^2}{3(2\pi)^3 m^2}\frac{1}{k_B T_e} d_0 \langle P_0 | P_0 \rangle \\ G_{12} = G_{21} &= \frac{2\hbar^2}{3(2\pi)^3 m^2} a_0 \langle P_0 | P_0 \rangle \\ G_{22} &= \frac{2\hbar^2}{2(2\pi)^3 m^2} k_B T_e a_1 \langle P_1 | P_1 \rangle \\ d_0 = \frac{\phi_{11}\langle P_0|P_0\rangle}{\Delta^{(1)}}, \qquad a_0 &= -\frac{\phi_{01}\langle P_1|P_1\rangle}{\Delta^{(1)}}, \qquad a_1 = \frac{\phi_{00}\langle P_1|P_1\rangle}{\Delta^{(1)}} \end{aligned} \tag{4}$$

where

$$\Delta^{(1)} = \phi_{00}\phi_{11} - \phi_{01}^2$$

and ϕ_{ij} denotes the sum of the Balescu–Lenard electron–electron and electron–ion collision operators. The above quantities are to be understood as a first polynomial approximation of the Chapman–Enskog treatment so that

$$\int d^3\boldsymbol{k} f^-(k^2) f^+(k^2) P_i(k^2) P_j(k^2) = \langle P_i | P_i \rangle \delta_{ij} \tag{5}$$

where

$$f^-(k^2) = \left[\exp\left(\frac{\mathscr{E}_k - \mu}{k_B T}\right) + 1\right]^{-1} \qquad \text{and} \qquad f^+ = 1 - f^-.$$

Since the electron–electron thermal conductivity K_{ee} has already been treated accurately (Lampe 1968), we will concentrate on the evaluation of the electron–ion conductivity K_{ei} for any degeneracy ($Z=1$, $T_e = k_B T/\mathscr{E}_F$):

$$K_{ei}^{(1)} = k_B \frac{\pi}{576} \frac{\hbar k_F}{m} \left(\frac{k_F}{\alpha r_s}\right)^2 \frac{T_e^4}{Z(\ln \Lambda^{-1} - \frac{1}{2})}$$

$$\times (21 F_{5/2} - 25 F_{3/2}^2)^2 \left(F_1 - \frac{5}{3} \frac{F_{3/2}}{F_{1/2}} F_0 + \frac{25}{18} \frac{F_{3/2}^2}{F_{1/2}^2} f^-(0)\right)^{-1}$$

where ($\alpha = \beta\mu$)

$$F_S = \int_0^\infty \frac{dx\, x^S}{e^{x-\alpha}+1}, \qquad \Lambda = (10)^{1/2} \frac{\lambda\!\!\!^-}{\lambda\!\!\!^-_D} \qquad \text{and} \qquad \lambda\!\!\!^- = \frac{\hbar}{2(m\mathscr{E}/\epsilon)^{1/2}}.$$

Equation (6) gives the expected limiting quantities:

$$K_{ei}^{(1)} = k_B \frac{\pi}{24} \frac{\hbar k_F}{m} \left(\frac{k_F}{\alpha r_s}\right)^2 \frac{T_e}{Z(\ln \Lambda^{-1} - \frac{1}{2})} \qquad \text{as } T_e \to 0 \qquad (7a)$$

$$K_{ei}^{(1)} = k_B \frac{75\pi^{3/2}}{13} \frac{\hbar k_F}{m} \left(\frac{k_F}{\alpha r_s}\right)^2 \frac{T_e^{5/2}}{\ln \Lambda^{-1} - \frac{1}{2}} \qquad \text{as } T_i \to \infty \qquad (7b)$$

in accord with the ionic weakly coupled limits of the classical (Spitzer 1962) and completely degenerate (Minoo *et al* 1976) results. The complete thermal conductivity is then $K^{(1)-1} = K_{ei}^{(1)-1} + K_{ee}^{(1)-1}$. Finally it should be noticed that the high-temperature classical Λ appears through $\lambda\sqrt{2} \to e^2/k_B T$ when $k_B T_e \leqslant 1$ Ryd.

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